Mortgage Options: Valuation, Risk Management, & Relative Value

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Andrew Lesniewski

`lesniewski@ellington.com`

Ellington Management Group

53 Forest Avenue

Old Greenwich, CT 06870

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- Muhammad Sattar

Mortgage options

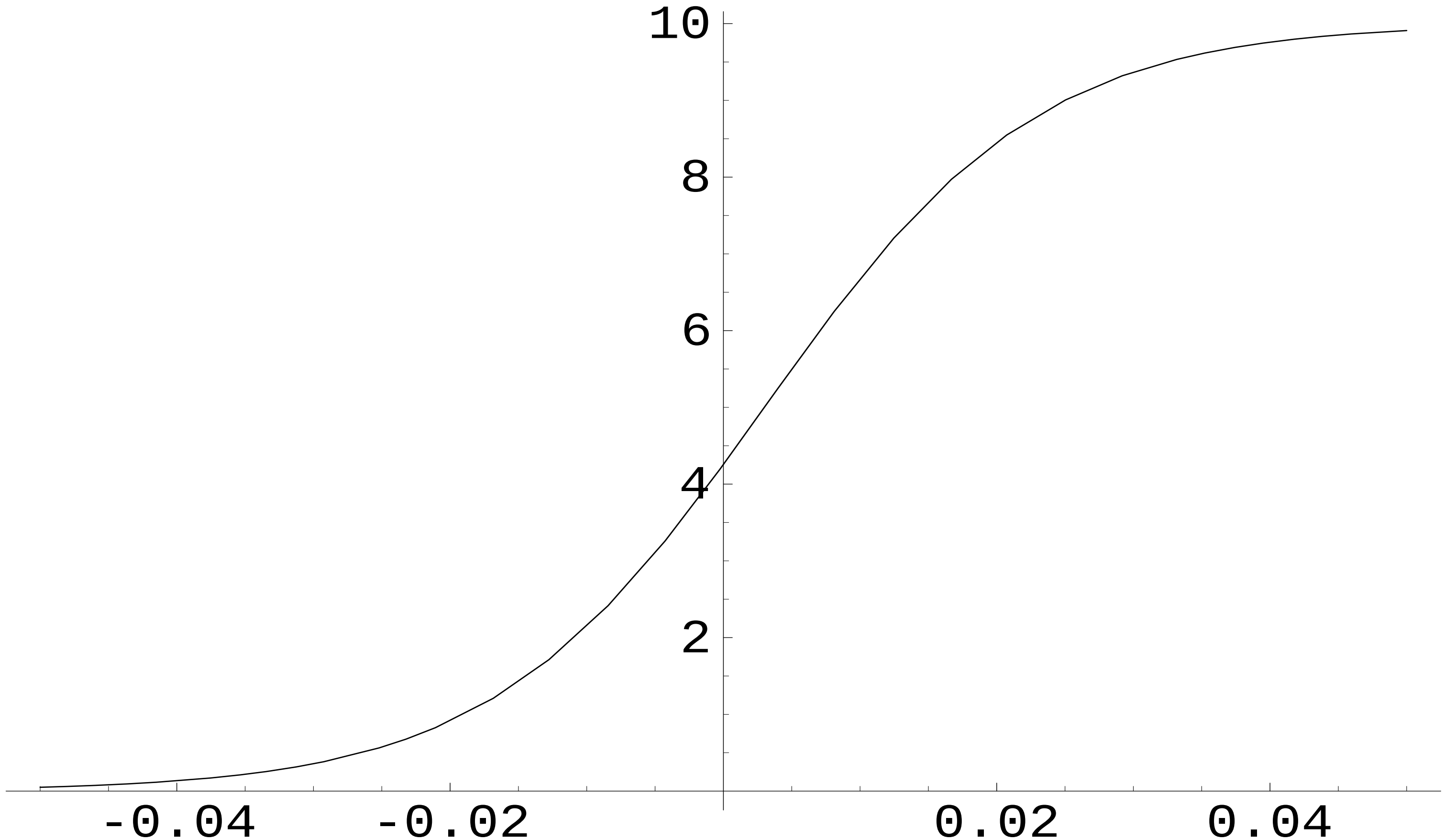
- Mortgage options = options on TBAs
- Expire 5 business days prior to the settlement of the TBA
- Standard strikes = {ATM, $\pm 1/2$, ± 1}
- Quotes available from Deutsche Bank and Goldman Sachs

Modeling mortgage options

- Duration of the TBA assumed a function $D(X)$ of the relative rate $X = M - C$, where C is the coupon on the TBA, and M is the current coupon.
- Option value depends on the normal volatility σ of X.
- $D(X)$ calibrated to the prepayment model implied duration.
- σ is implied by the options market.

Duration of a TBA

We parameterize the duration of a TBA



Duration of a TBA

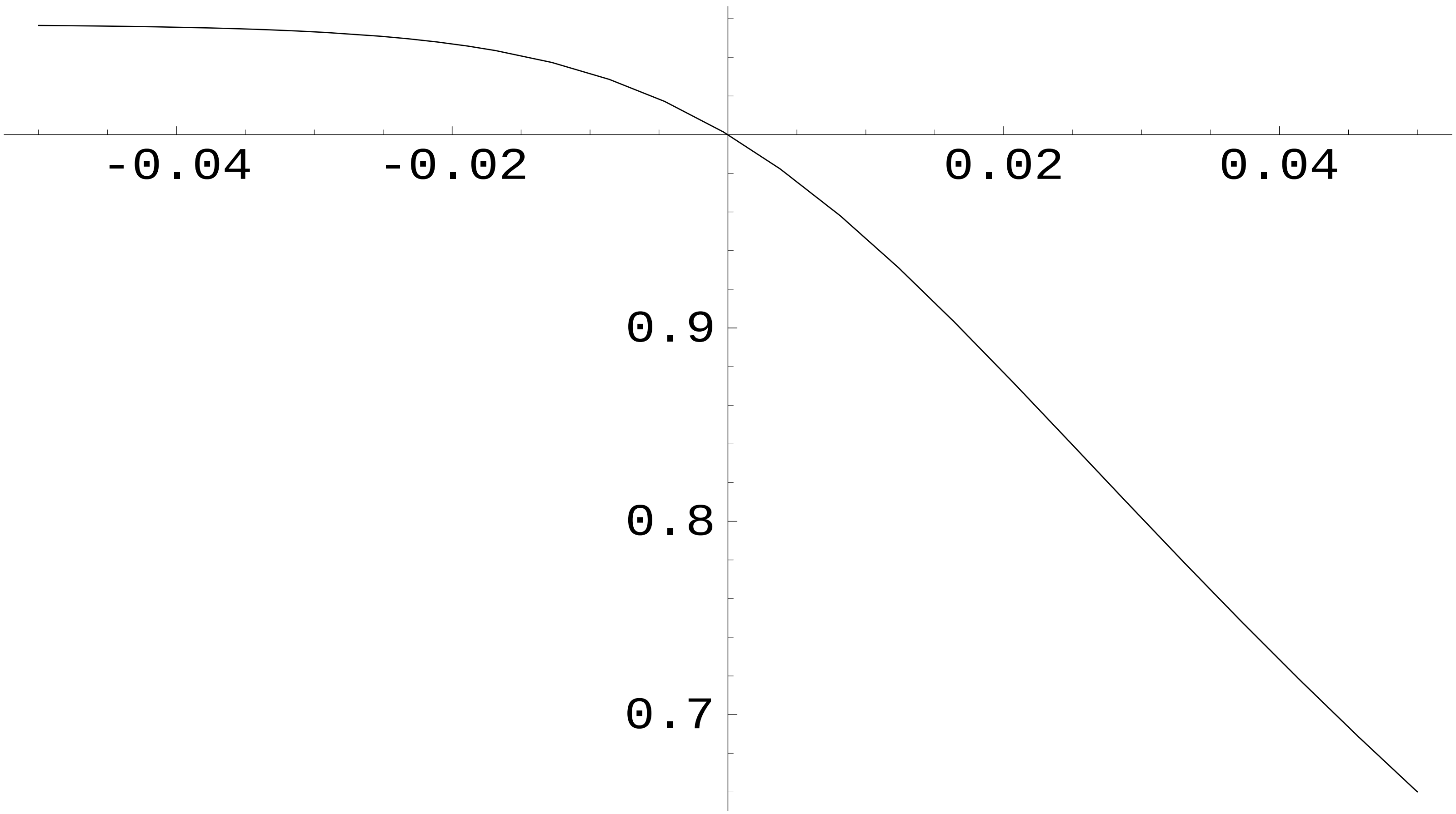
by means of a logistic function of the relative rate X:

$$D(X) = L + (U - L)\ \frac{1}{1 + e^{-\kappa(X-\Delta)}}$$

The parameters L, U, κ and Δ are determined by fitting to durations computed from the prepayment model.

Price of a TBA

This produces the following shape of the price function of the TBA:



Price of a TBA

which is explicitly given by

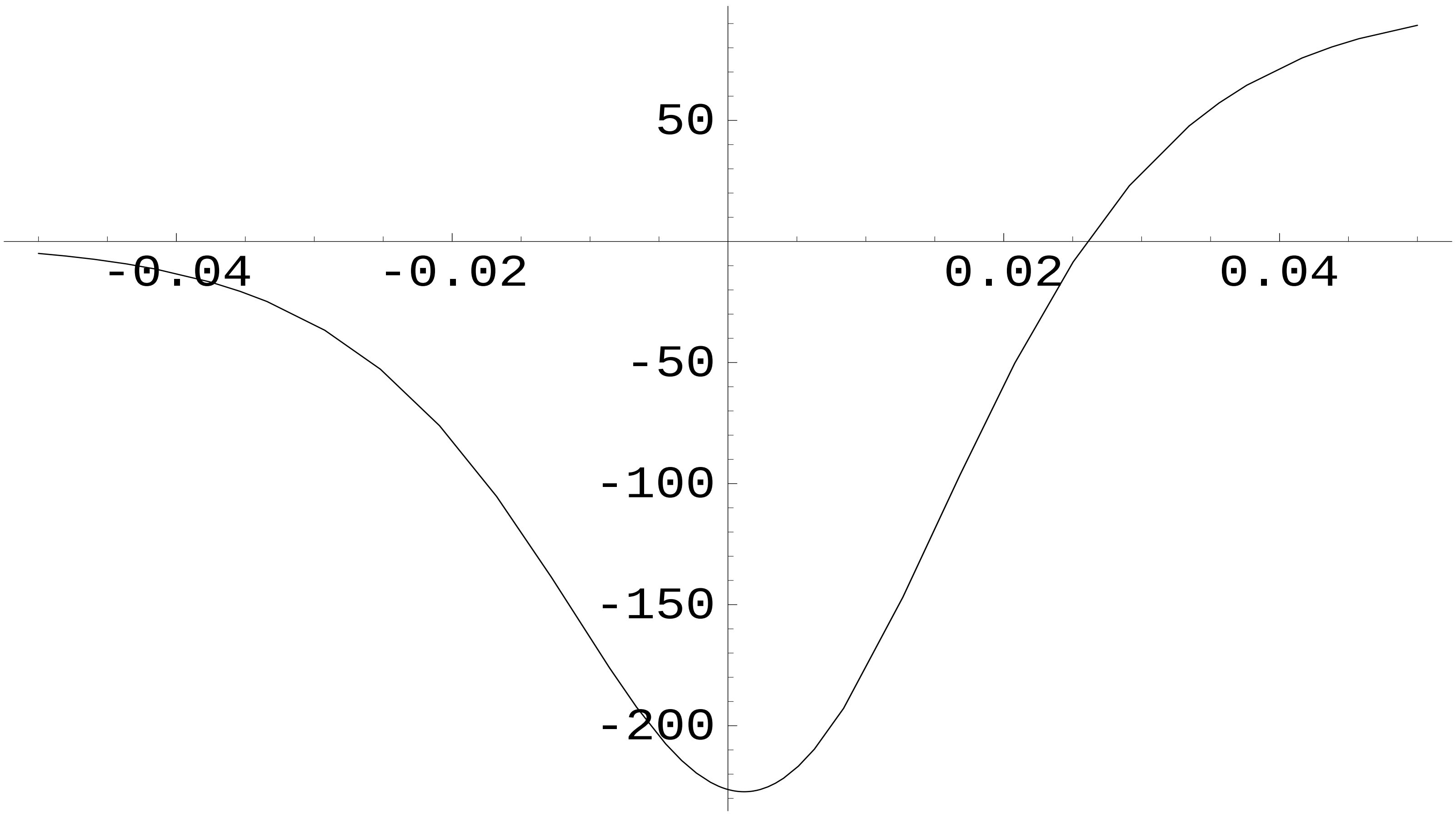
$$P(X) = \mu \ \exp\left(-\frac{(L+U)\,X}{2}\right) \ \left\{\frac{\cosh\frac{\kappa\Delta}{2}}{\cosh\frac{\kappa(X-\Delta)}{2}}\right\}^{\frac{U-L}{\kappa}}$$

This function is the key input to the option model.

The value of $\mu = P(0)$ is determined so that the distribution of prices is consistent with the options market, and is close to par.

Convexity of a TBA

The convexity of the TBA has the shape:



Convexity of a TBA

and is explicitly given by the following expression:

$$\Omega\left(X\right) = D\left(X\right)^2 - \frac{\kappa\left(U - L\right)}{4}\ \frac{1}{\cosh^2\frac{\kappa(X-\Delta)}{2}}$$

Calibrating duration function parameters

- Generate macro rates scenarios (say, ± 25 bp, ± 50, ± 140, ...) and compute the corresponding durations of the TBA
- Fit the function $D(X)$. For the Fannie 5.0, the calibrated parameters are:

$$L = -0.677$$

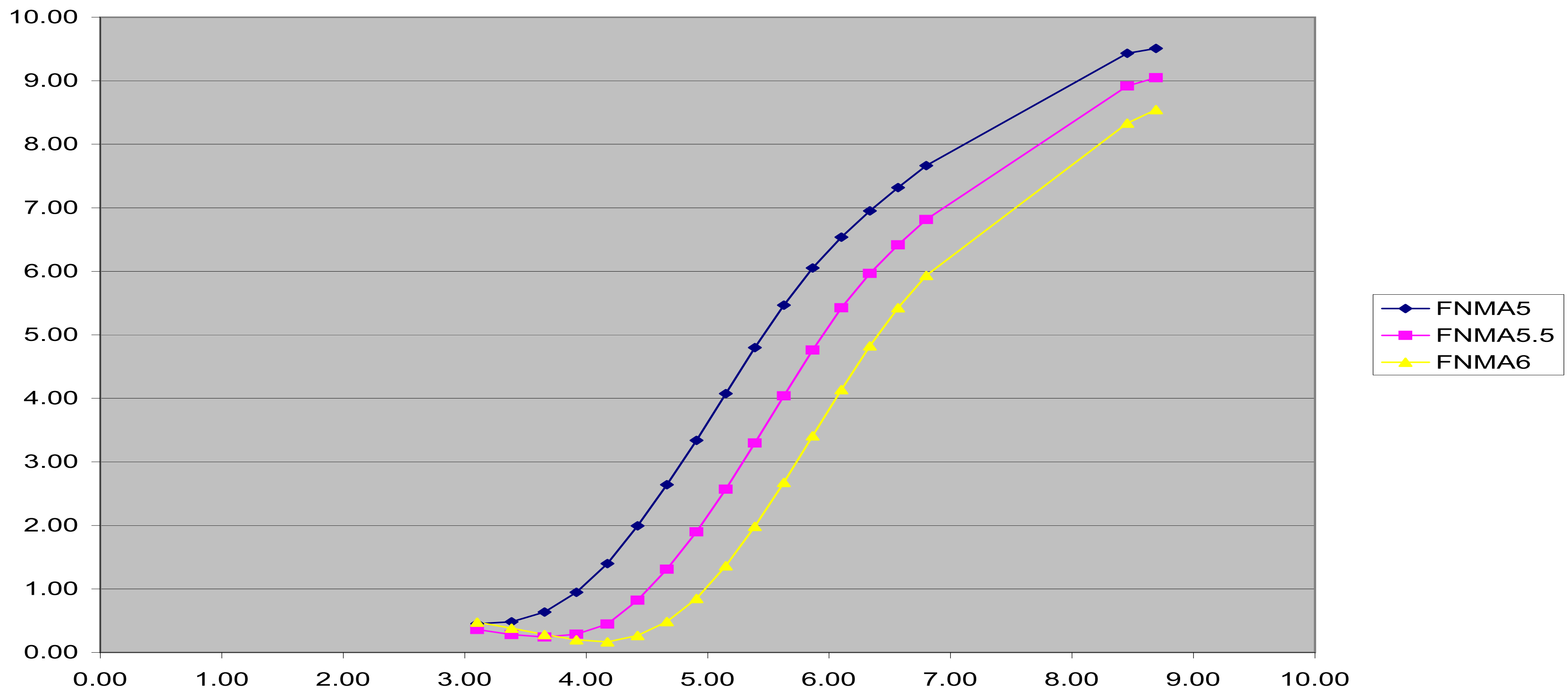
$$U = 9.679$$

$$\Delta = 0.00344$$

$$\kappa = 108.624$$

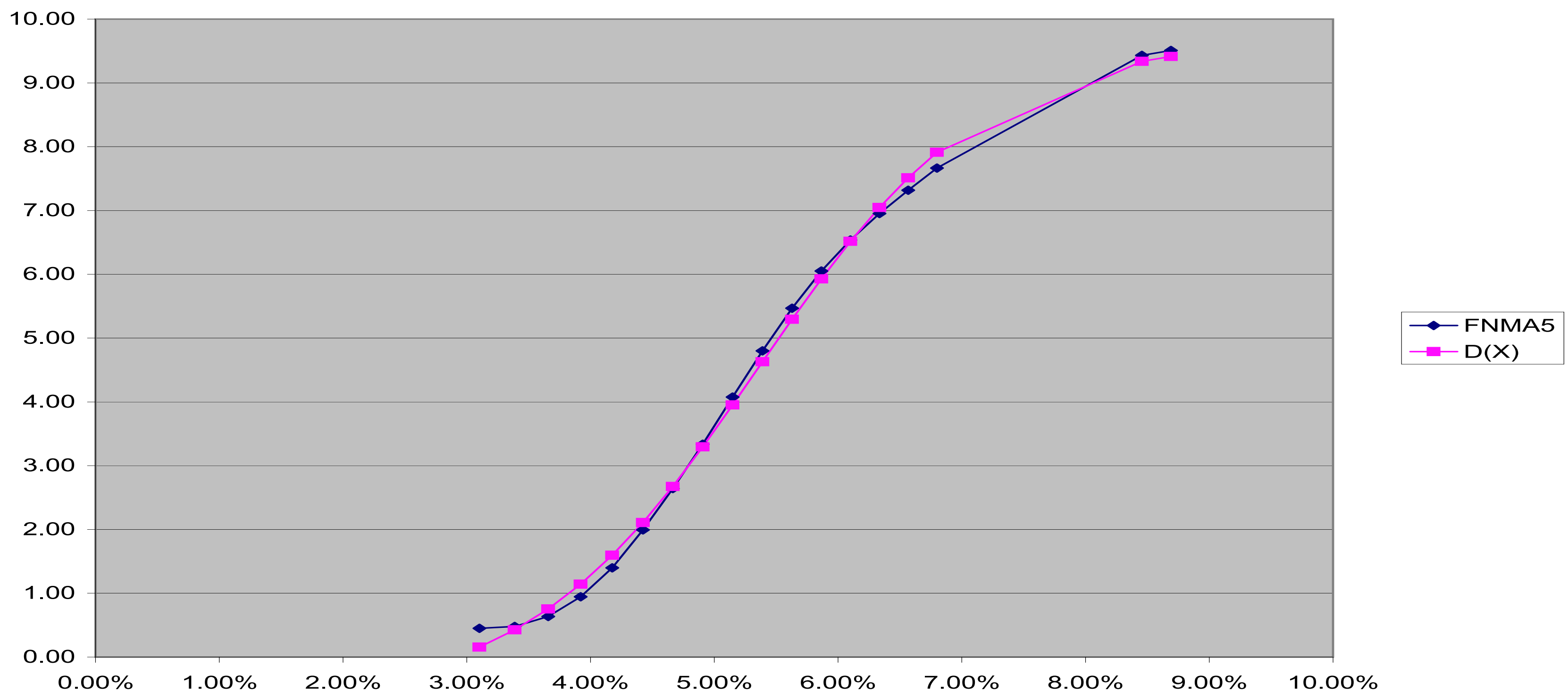
Calibrating duration function parameters

The graph below shows the durations of the TBAs of different coupons as a function of the current coupon.



Calibrating duration function parameters

On this graph, the TBA duration is compared to the calibrated logistic function $D(X)$.



Mortgage rate process

We assume that the relative rate follows a normal process

$$dX(t) = \sigma dW(t),$$

$$X(0) = M - C,$$

where $W(t)$ is a Brownian motion. The solution

$$X(t) = M - C + \sigma W(t)$$

is implemented by a Monte Carlo simulation.

Mortgage option price

Calls and puts struck at K and expiring at T are valued according

$$\mathcal{C} = Z(T)\,\mathsf{E}\left[\max\left(P(X) - K,\ 0\right)\right]$$

$$\mathcal{P} = Z(T)\,\mathsf{E}\left[\max\left(K - P(X),\ 0\right)\right]$$

Here, $Z(T)$ denotes the discount factor.

The valuation formulas do not lead to closed form expressions, and the evaluation of the expectations requires Monte Carlo simulations. The simulations lead to fast and accurate results.

Implied current coupon volatility

We can now easily calibrate the volatility σ by requiring that

- the price of the TBA, and
- the premium on the option

match the market values. This is done in two steps:

- Choose the parameter $\mu = \mu(\sigma)$ so that the mean of the price distribution is the market price of the TBA.
- Choose σ to match the option premium.

Currently (as of September 17), $\sigma \simeq 265$ bp.

Implied current coupon volatility

Specifically, here is the September 17 snapshot of the options market (November expires):

FNCL5.0	K	-1	$-1/2$	ATM	$+1/2$	$+1$
$99-016$	σ	$1-07$	$1-12+$	$1-185$	$1-095$	$1-013$

FNCL5.5	K	-1	$-1/2$	ATM	$+1/2$	$+1$
$100-26+$	σ	$0-273$	$1-002$	$1-057$	$0-28+$	$0-201$

FNCL6.0	K	-1	$-1/2$	ATM	$+1/2$	$+1$
$102-012$	σ	$0-172$	$0-212$	$0-261$	$0-16+$	$0-08+$

Implied current coupon volatility

and the implied current coupon volatilities (in basis points) are:

FNCL5.0

K	-1	$-1/2$	ATM	$+1/2$	$+1$
σ	262.5	263.4	264.2	265.7	267.4

FNCL5.5

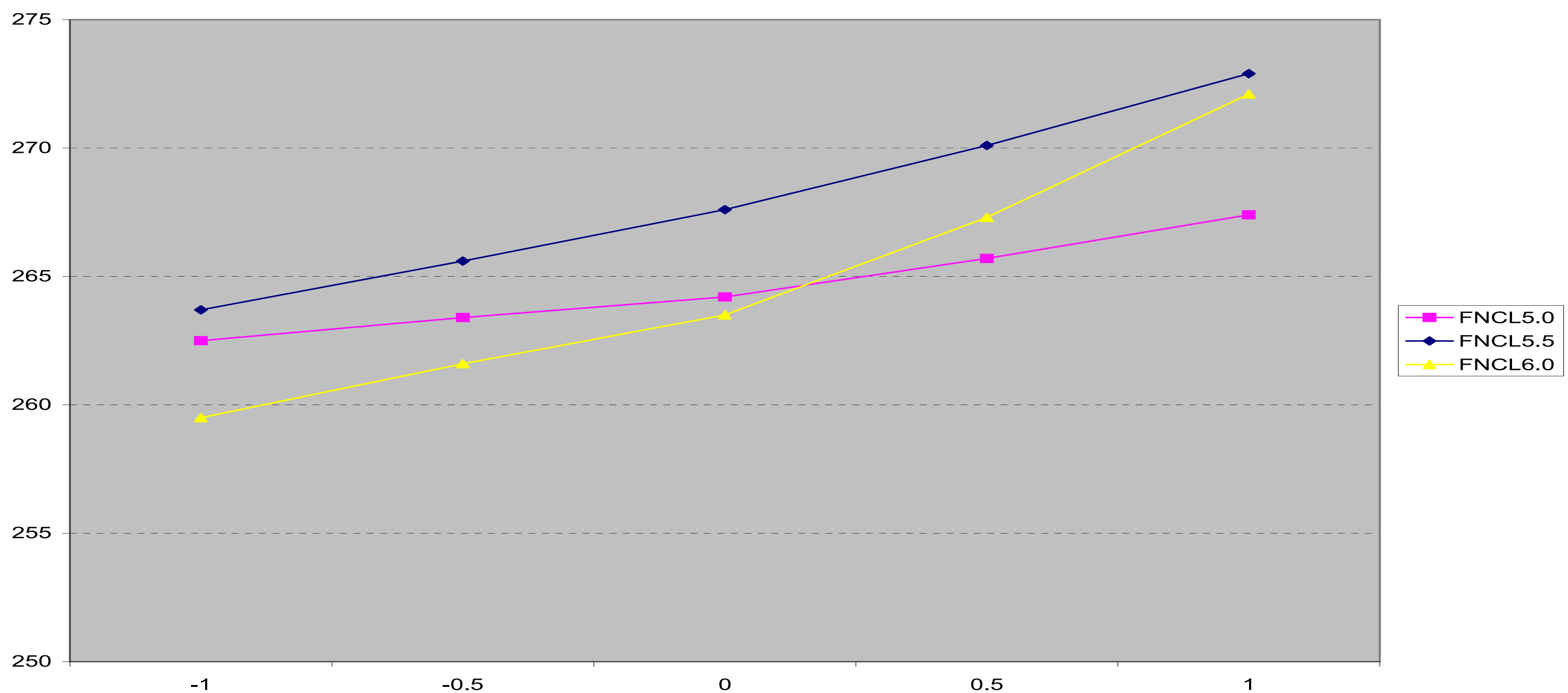
K	-1	$-1/2$	ATM	$+1/2$	$+1$
σ	263.7	265.6	267.6	270.1	272.9

FNCL6.0

K	-1	$-1/2$	ATM	$+1/2$	$+1$
σ	259.5	261.6	263.5	267.3	272.1

Volatility smile

Implied volatility exhibits smile:



Relative value

The stochastic model allows to asses richness / cheapness of current coupon volatility:

- Versus realized current coupon volatility ($\simeq 235$ bp as of September 17).
- Versus suitable swaption volatilities.

Risk characteristics of mortgage options

Mortgage options are options on convex assets (prepayment options). Their risk is a composition of optionalities.

- Delta

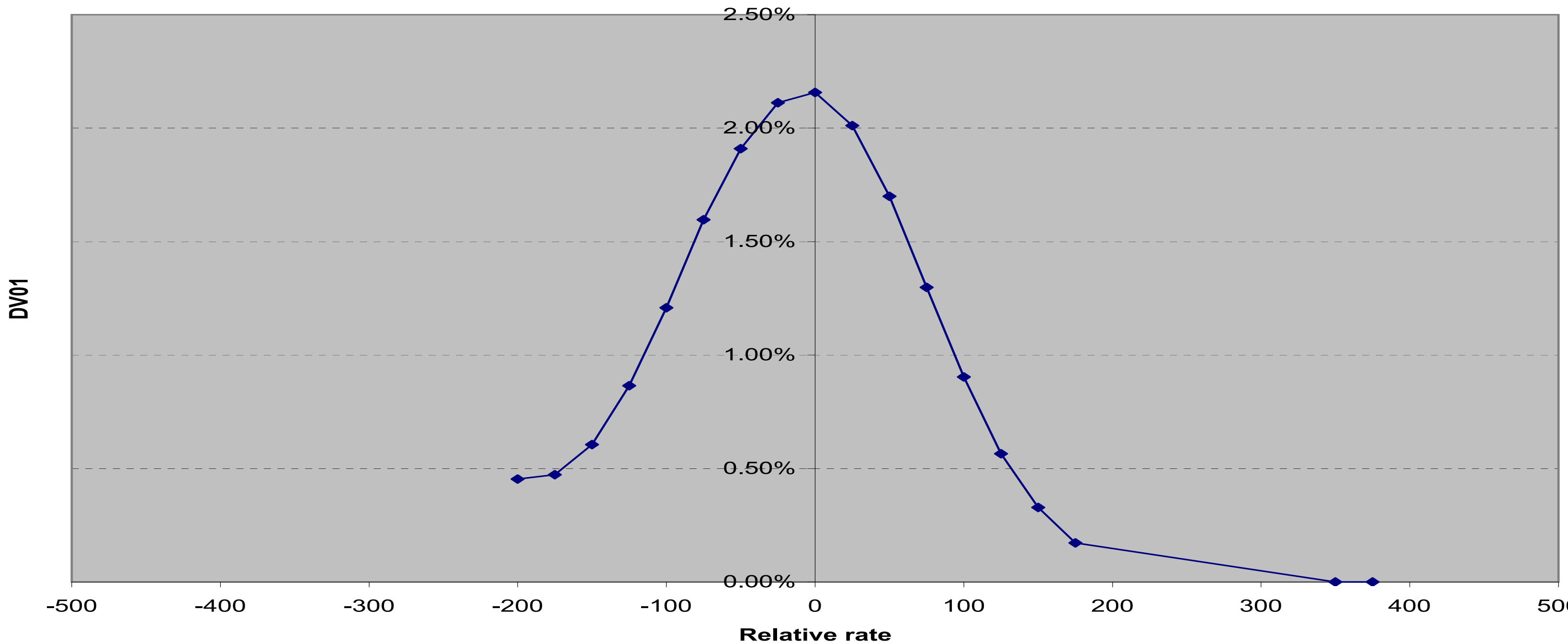
$$\Delta = \Delta_{BS} \times \Delta_{TBA}$$

- Gamma

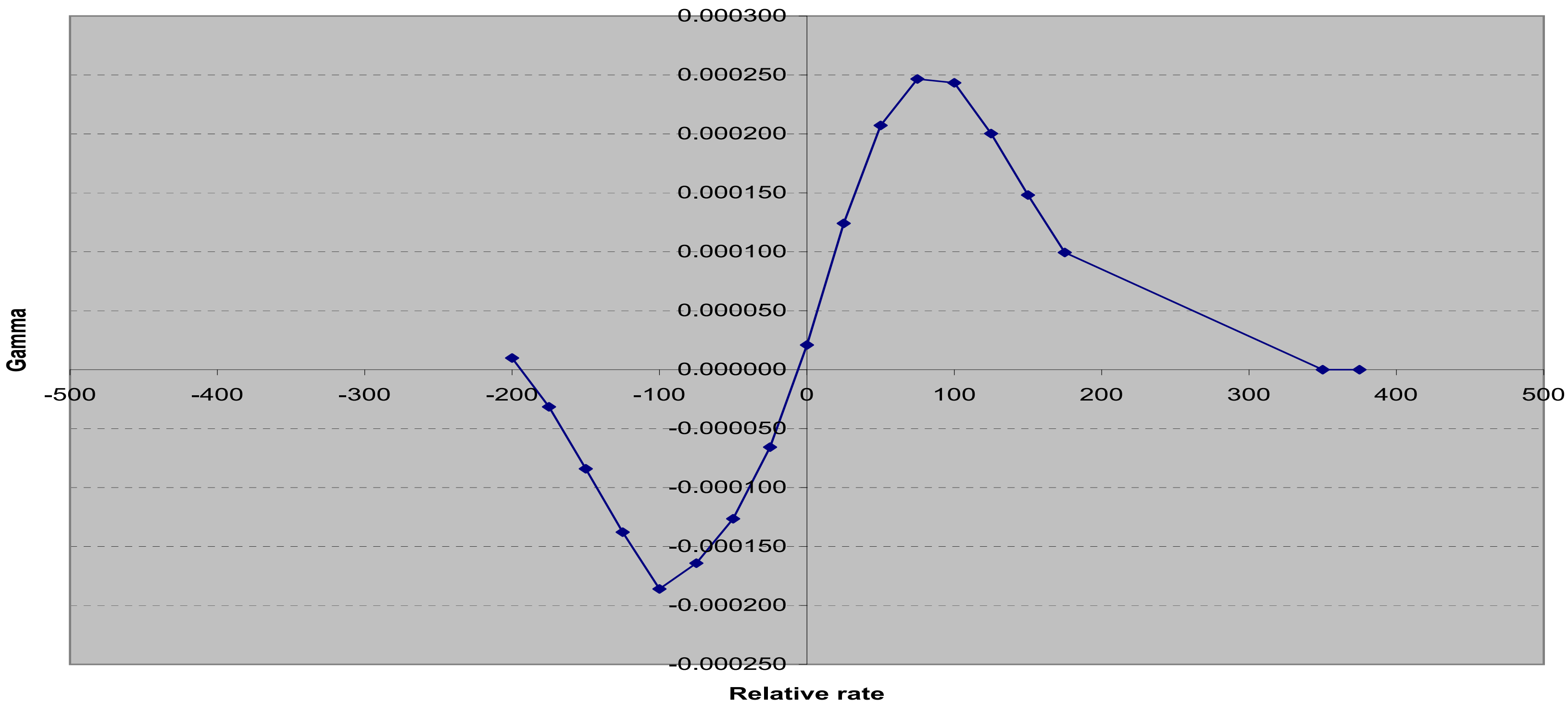
$$\Gamma = \Gamma_{BS} \times (\Delta_{TBA})^2 + \Delta_{BS} \times \Gamma_{TBA}$$

- ...

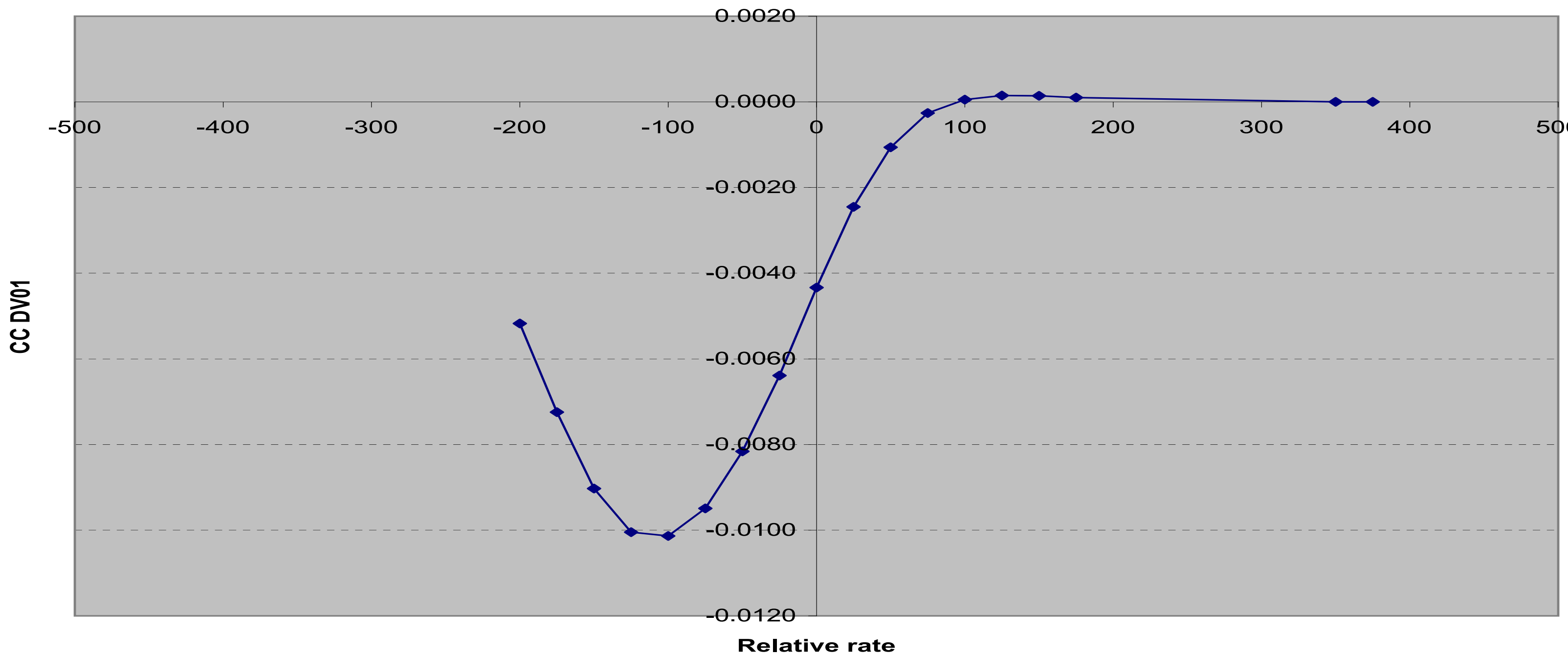
Delta risk of a mortgage call



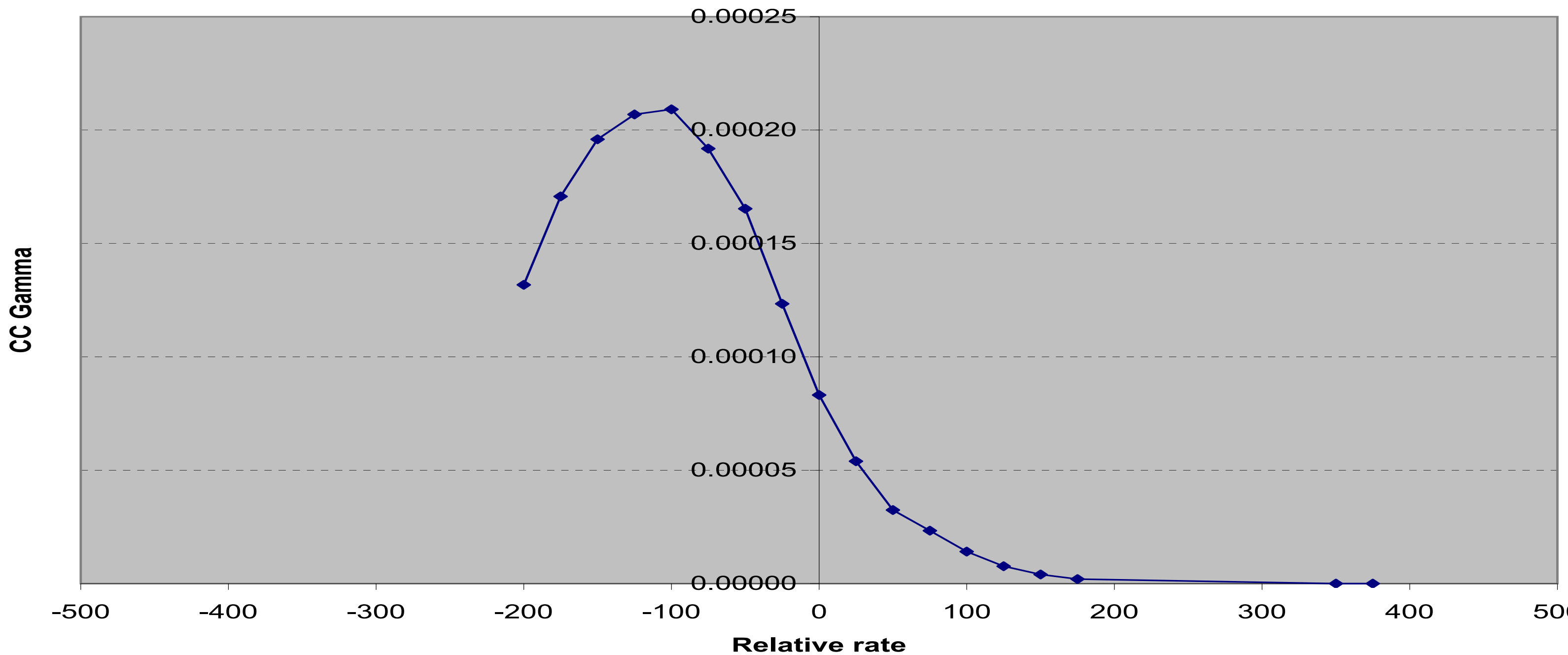
Gamma risk of a mortgage call



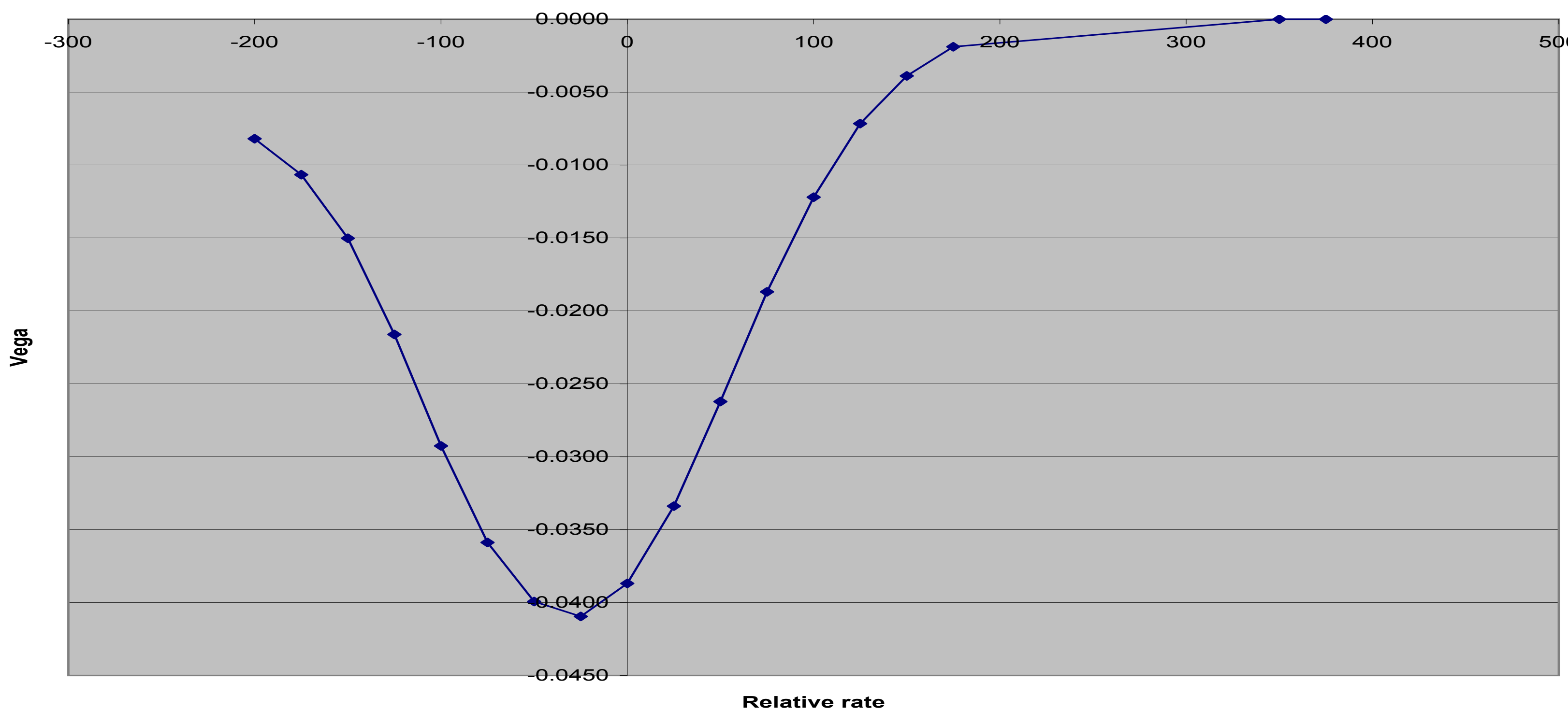
CC delta risk of a mortgage call



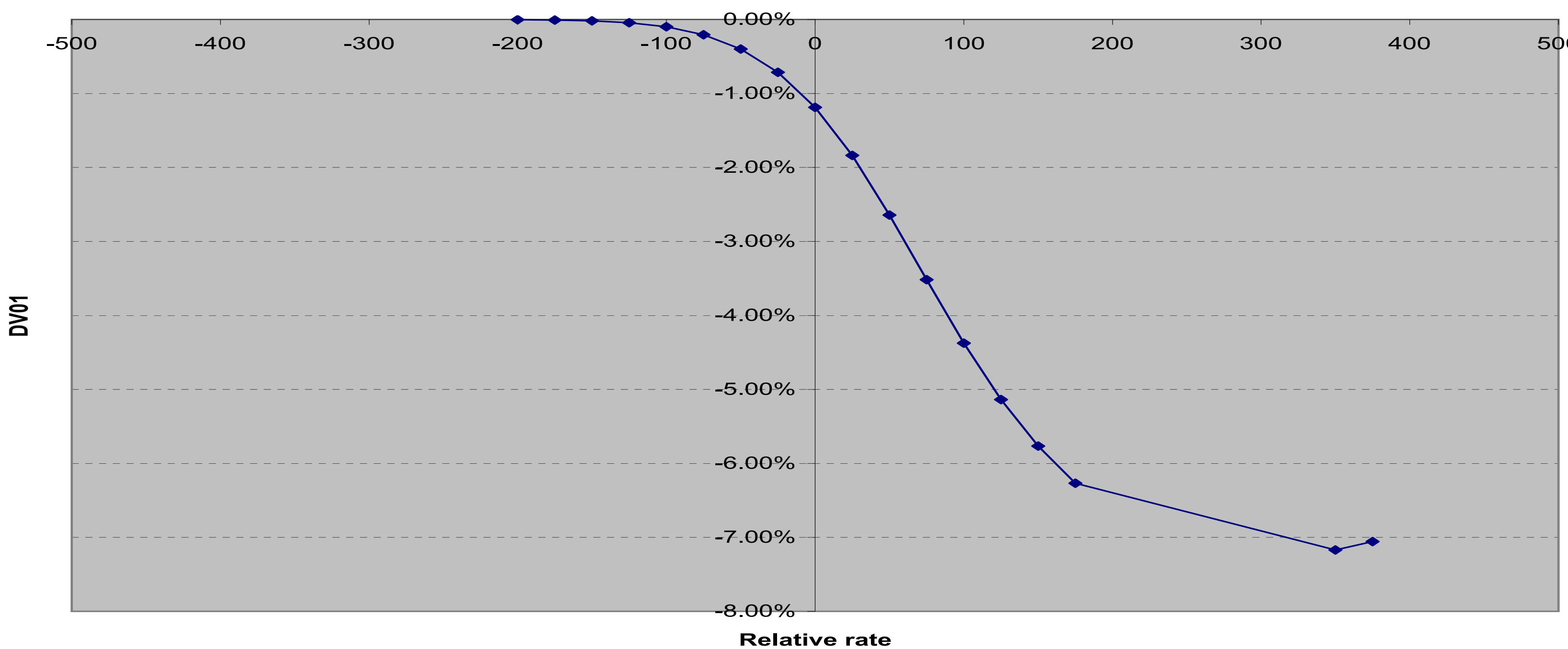
CC gamma risk of a mortgage call



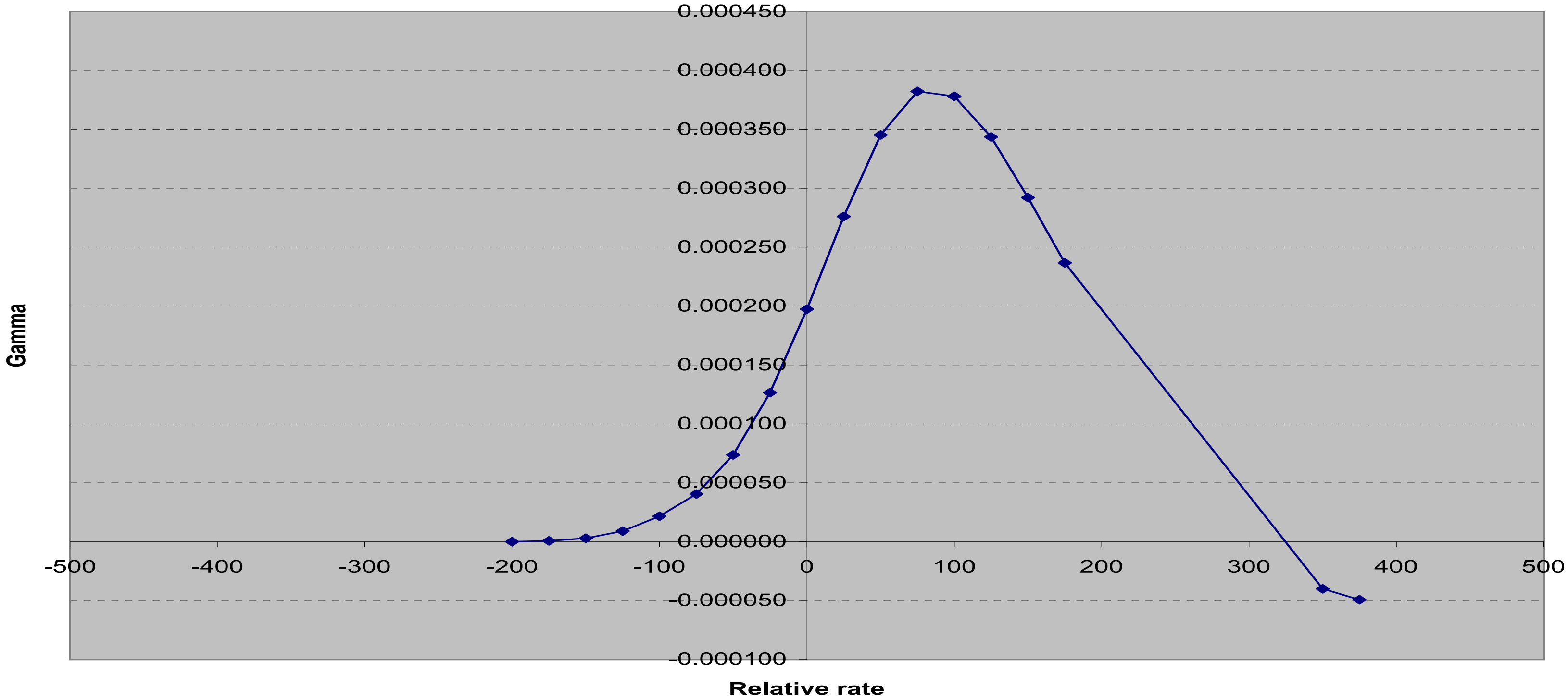
Swaption vol vega risk of a mortgage call



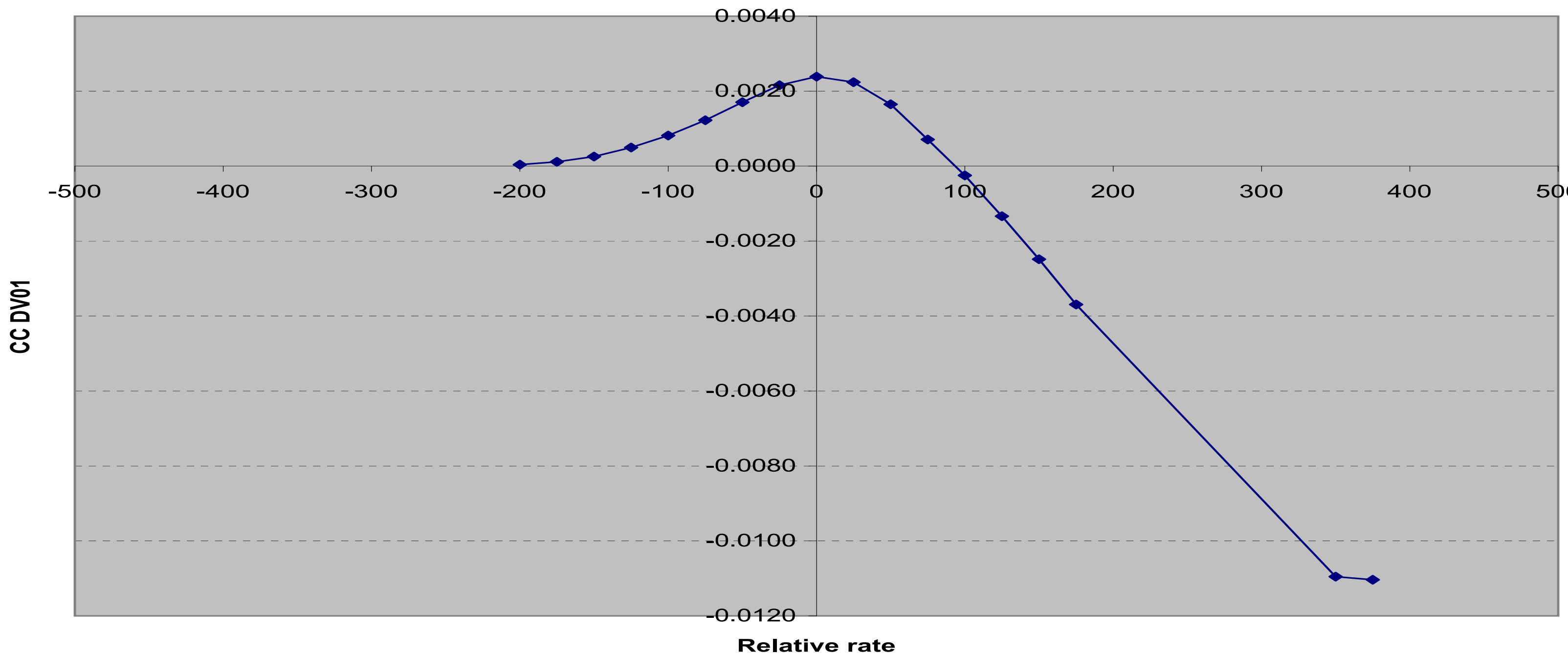
Delta risk of a mortgage put



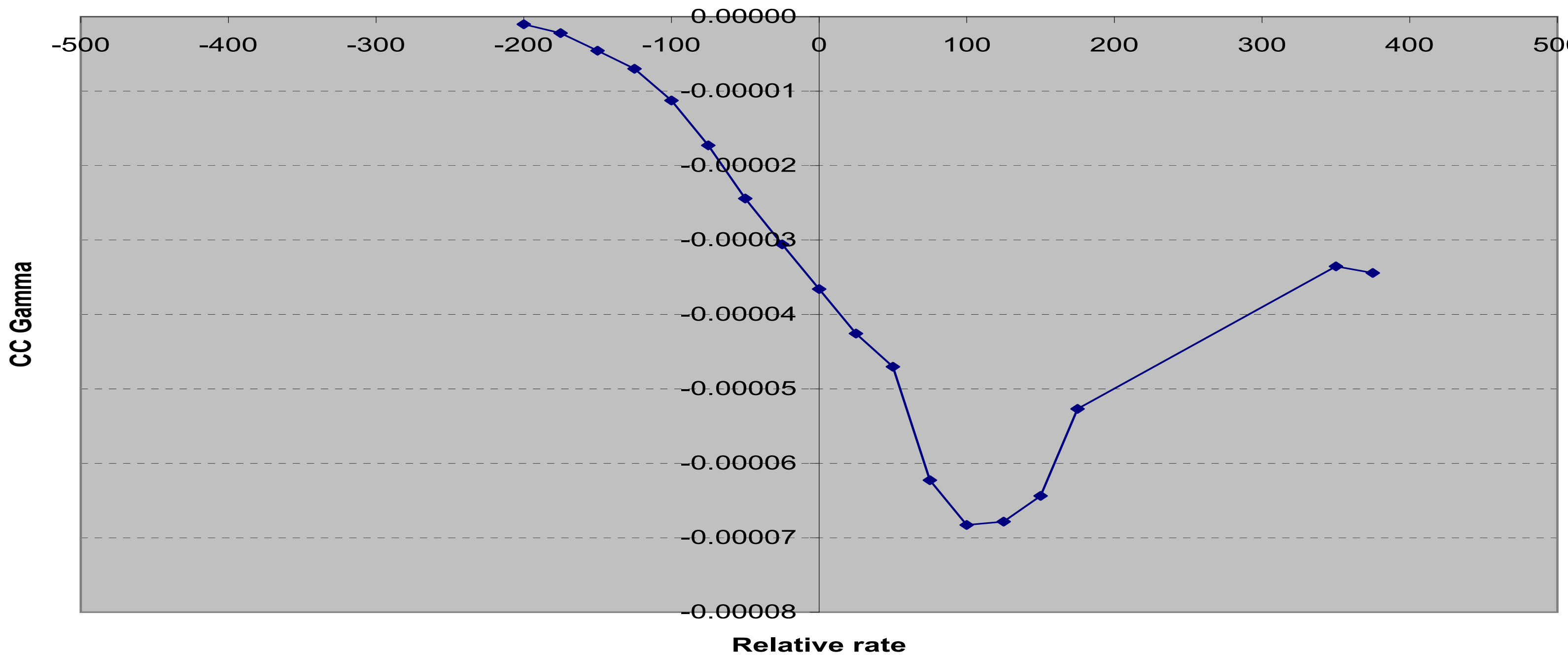
Gamma risk of a mortgage put



CC delta risk of a mortgage put



CC gamma risk of a mortgage put



Swaption vol vega risk of a mortgage put